Chapter 7

Strategy #6

Formal Proofs

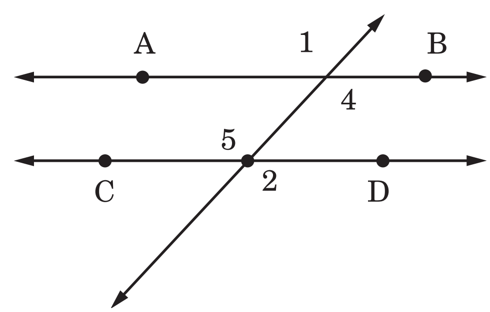
Strategy #6: Write formal proofs in 2-column form.

✓ In the second half of Strategy 5 you were introduced to the 2-column proof. This is a formal, organizational tool for proof-writing. It allows you to make statements in one column and justifications (reasons) in the second column. By lining up the statements with the reasons which defend them, it is easy for anyone to read your proof and understand what you are saying.

✓ The strategies in this workbook have been designed to help you develop the skills and thinking patterns you need to write formal proofs. As you work on these, and future, proofs, think back on the exercises in this workbook. If you are stuck in a proof, use the methods of the workbook to help you:

- Stop thinking about what you're trying to prove and try brainstorming about the figure itself.
- Work backwards from what you are trying to prove. Ask yourself, "What would I need to know to say this?"
- Write steps for different parts of the proof and then try to fit them together.
- Allow yourself to make mistakes, run into 'dead ends' with lines of reasoning; eventually you'll find a line of reasoning that leads to what you are trying to prove.
- Sometimes writing a paragraph proof first can lay out the thinking before switching over to a formal 2-column proof (sometimes the 2-column proof is easier and the paragraph proof comes later…it depends on the proof and the person).

For Example *Write a proof in 2-column form.*
Given: $\angle 1 \cong \angle 2$.
Prove: $\overline{AB} \parallel \overline{CD}$.



Think about the problem before you try to write the proof: $\angle 1$ and $\angle 2$ don't have a direct relationship. However, they both have vertical angles which are shown in the figure. $\angle 1$ and $\angle 4$ form a pair of vertical angles, and $\angle 2$ and $\angle 5$ form a pair of vertical angles. Since we know that pairs of vertical angles are congruent, we can use that relationship, combined with the transitive property, to show that $\angle 4$ and $\angle 5$ are congruent. Since $\angle 4$ and $\angle 5$ are alternate interior angles and since we will be able to show that they are congruent, we will be able to conclude that these lines are parallel.

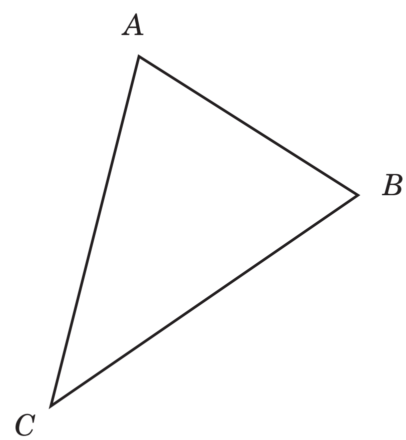
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1 \cong \angle 4$; $\angle 5 \cong \angle 2$	2. Vertical angles are congruent.
3. $\angle 4 \cong \angle 5$	3. Substitution
4. $\overline{AB} \parallel \overline{CD}$	4. Alternate interior angles

Chapter 1

Triangle Basics

What Is a Triangle?

•***A triangle has 3 sides which meet in three vertices.***
Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$. These are read as "segment *AB*" or "side *AB*."
Vertices (singular = vertex): *A*, *B*, *C*. These are read as "*A*" or "vertex *A*."



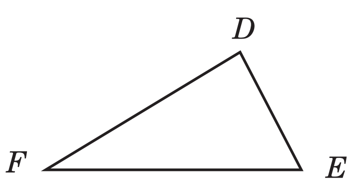
•***The angles of a triangle always add up to 180º.***
Angles: $\angle A$ (or $\angle CAB$, or $\angle BAC$), $\angle B$ and $\angle C$.
These are read "angle *A*" or "angle *CAB*."
•If you want to refer to the measure of the angle, and not its name, use the notation: $m\angle A$. This is read "the measure of angle *A*."

•***When naming a triangle, you can name the 3 vertices in any order***. The letters are preceded by the symbol for a triangle.
Names for this triangle: ΔABC, ΔBAC, ΔCAB, ΔACB, ΔBCA, ΔCBA.
These are read as "triangle *ABC*."

Practice

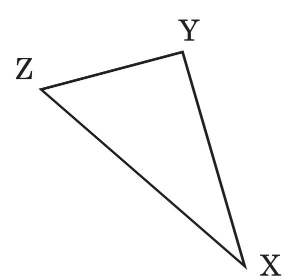
Name these triangles and name each of their angles.

Example



Options for naming the triangle above include: ΔDEF, ΔFED and ΔEDF, among others. Options for naming the angles include: $\angle D$, $\angle F$ and $\angle E$ or $\angle EDF$, $\angle EFD$ and $\angle$DEF.

1.



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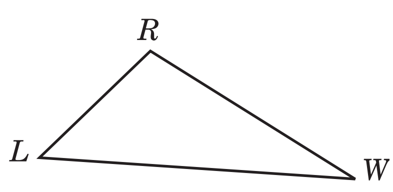

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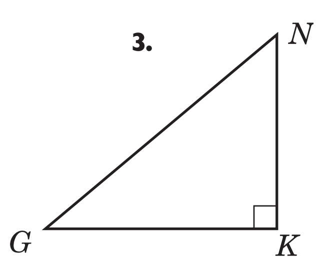

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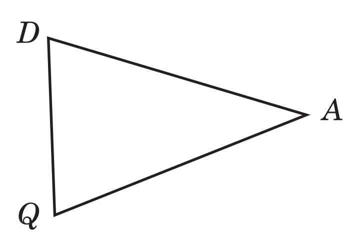

Chapter 5

Special Segments in Triangles

Each of the constructions in this section involves the basic constructions you have been using throughout this book. The median, angle bisectors, perpendicular bisectors and altitudes are unique because they are important elements of triangles in the study of geometry.

Angle Bisectors

There are 3 angles in a triangle, one at each vertex. Therefore, there are three angle bisectors in every triangle.

To construct the angle bisector for the angle whose vertex is at *A*:

1. Ignore side $\overline{BC}$. Construct the angle bisector for $\angle BAC$ as you learned previously.

Try this construction. *Construct the angle bisectors of the other 2 angles of this triangle.*